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Step-by-Step Function Analysis: ChatGPT's Performance and Its Role in Supporting Student Understanding

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ABSTRACT

This study examines the potential of ChatGPT as a complementary tool for teaching function analysis, with a particular focus on economics students. While traditional computer algebra systems provide correct results, ChatGPT adds value through step-by-step explanations and intuitive analogies. The model was tasked with analyzing the function (f(x)=x^4-10x^2+9), producing both formal mathematical solutions and accessible explanations. Results show that the AI can accurately determine zeros, extrema, and inflection points while linking these outcomes to economic decision-making, such as cost minimization and profit maximization. This enables students to view function analysis not only as a technical procedure but also as a tool for real-world applications. However, the findings emphasize that AI should be regarded as a pedagogical enhancement rather than a replacement for the teacher, whose role remains essential in providing guidance, motivation, and ethical oversight.

Introduction

The use of artificial intelligence (AI) in education has been explored for several decades. However, recent technological progress has introduced possibilities that are now reshaping traditional approaches to teaching and learning. In the 1970s and 1980s, the first intelligent tutoring systems were designed to bring computational technologies into classrooms [1, 2]. These early systems could handle certain algorithmic tasks, but they were far from acting as genuine interactive learning partners. Over the following decades, programs were gradually developed to support more personalized instruction [3–5]. By the 2010s, adaptive learning systems and learning analytics became available, offering real-time feedback and responses to students' performance [6, 7].

The release of ChatGPT in November 2022 marked a turning point in this trajectory. As a generative language model, ChatGPT can produce coherent, human-like text in multiple languages and across diverse contexts [8]. Within a short time, it became one of the most debated technologies in higher education. Although its potential to enhance learning has been widely acknowledged, it has simultaneously raised significant ethical and pedagogical challenges [9].

Mathematics is a particularly relevant area for such discussions. On the one hand, mathematical problem-solving often involves sequences of algorithmic steps—tasks that computational systems can carry out reliably. On the other hand, it requires logical reasoning and abstract thinking, which remain more challenging for large language models. Function analysis illustrates this duality especially well. Students must not only apply formal techniques, such as differentiation, but also interpret the results. Where are the zeros of the function? Where does it increase or decrease? At which points are the maxima or minima, and what do these results mean in practical terms?

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This is especially important in economics, where the outcomes of function analysis have clear applications: identifying the minimum cost, the maximum revenue, or the maximum profit are central tasks for decision-making. The present study investigates whether ChatGPT can not only perform the mathematical steps of function analysis correctly but also explain them in ways that are accessible to students with varying levels of expertise, from secondary school learners to university students. By examining both performance and explanatory clarity, we aim to evaluate the role that ChatGPT may play in supporting mathematics learning in economics education.

Literature Review

The relationship between AI and education has evolved through a long and complex trajectory, with early systems designed primarily to support algorithmic learning processes and to automate specific elements of instruction [1, 2]. However, the flexibility of these systems was limited, which meant that they could only operate effectively in standardized learning environments. During the 1990s and early 2000s, research shifted toward more personalized approaches to learning [3–5]. By the 2010s, learning analytics and adaptive systems had become central, offering real-time feedback on student performance [6, 7].

The introduction of ChatGPT marked a new stage in this evolution. Large language models (LLMs) extend far beyond the capabilities of earlier educational software: they are able to generate coherent text that closely mirrors human expression, even at a stylistic level [8]. This capacity has introduced new dilemmas in education. Some institutions perceived ChatGPT as a threat and attempted to restrict its use [10, 11], while others identified pedagogical opportunities, such as fostering critical thinking or simplifying complex concepts [12, 13].

In mathematics education, the research evidence has been mixed. Liu and colleagues [14] demonstrated that while ChatGPT's reasoning abilities have improved, errors still occur when facing novel or complex problem types. Cruz et al. [15] found that, in the context of calculus tasks, ChatGPT's solutions often contained inaccuracies; yet, under teacher guidance, it could still support student learning effectively. Similarly, Udías and co-authors [16] showed through an empirical study that ChatGPT-4 outperformed average examinees in statistics and probability problems on Spanish university entrance examinations, while performing less effectively in algebra and analysis.

Overall, the literature indicates that ChatGPT's mathematical performance is highly dependent on the type of problem. Its greatest pedagogical value may not lie in flawless computation, but rather in its dual explanatory capacity: providing formal mathematical reasoning while also offering intuitive, everyday analogies that facilitate student understanding. This duality is particularly significant in the teaching of function analysis, which simultaneously requires rigorous formal reasoning and accessible interpretation.

Methodology

For the purposes of this study, we selected a specific task in function analysis: the function $f(x)=x^4-10x^2+9$. This type of exercise is a typical example of the problems that undergraduate students frequently encouter in introductory courses in economics and applied mathematics. The experiment involved asking ChatGPT5 to produce two types of explanations: first, a formal mathematical analysis, and second, an interpretation expressed in everyday, non-technical language.

To perform the mathematical computations, ChatGPT5 relied on the Python programming language in combination with established libraries such as SymPy, Matplotlib, and Pandas. The use of these tools minimized the likelihood of computational errors and ensured that the AI's results could be compared directly with those generated by conventional computer algebra systems.

For the non-technical explanations, ChatGPT drew upon the linguistic capabilities of its large language

model. During training, the model had been exposed to a broad range of educational resources, including textbooks, explanatory texts, and analogies frequently used in teaching. As a result, ChatGPT was able to articulate the same concepts on multiple levels:

- in precise mathematical language, and
- through accessible, image-based metaphors (e.g., hills, valleys, bowls, domes).

This dual approach enabled the demonstration of ChatGPT's ability to bridge the gap between formal symbolic reasoning and intuitive conceptual understanding. Rather than conducting empirical data collection, the study focused on a qualitative examination of ChatGPT's outputs, assessing both the accuracy of the mathematical reasoning and the clarity of the explanations.

The evaluation thus centered on two key criteria:

1. *Mathematical correctness* – whether ChatGPT could accurately execute the formal steps of function analysis, including finding zeros, critical points, intervals of monotonicity, convexity, and inflection points.
2. *Pedagogical clarity* – whether the AI could explain these results in a way that would be understandable to non-specialist learners, particularly students in economics, where function analysis has direct applications (e.g., cost minimization, revenue maximization, profit optimization).

Results

The solution generated by ChatGPT for the selected function analysis task proved to be mathematically correct. The system successfully determined the domain of definition, identified the symmetry of the function, computed the zeros, and accurately located the critical points corresponding to local maxima and minima. Furthermore, it was able to identify the intervals of monotonicity, determine the inflection points, analyze the intervals of convexity and concavity, and establish the correct range of the function. In all of these formal components, the output of ChatGPT aligned with what would be expected from a traditional step-by-step mathematical analysis.

What made the output particularly valuable, however, was not merely the correctness of the calculations but the accompanying explanations. Alongside the use of precise mathematical language, ChatGPT provided intuitive analogies designed to make the abstract results more accessible to learners. For example, local maxima were described metaphorically as *mountain peaks*, while minima were presented as *valleys.* Similarly, the transition in curvature at inflection points was explained through imagery such as the shift from a *dome* to a *bowl* (see Figure 1 and 2*).*

These metaphors represent more than stylistic choices; they serve as cognitive tools that help bridge the gap between abstract formalism and intuitive understanding. In mathematics education, such dual-layered explanations are particularly beneficial for learners who may struggle with purely symbolic reasoning. By framing maxima as peaks and minima as valleys, ChatGPT invited students to visualize the function's behavior in familiar, everyday terms. This dual explanatory strategy—formal rigor combined with accessible metaphor—underscores ChatGPT's potential as an educational support tool, particularly in disciplines such as economics where the interpretation of mathematical results carries direct practical significance.

Dual view — full, detailed table

Formal mathematics (step by step)	Plain-language explanation (what/why)
0. Problem: Analyze $f(x) = x^4 - 10x^2 + 9$ (a 4th-degree polynomial).	This rule takes any input x and outputs a number $f(x)$. It's a standard polynomial—no square roots or denominators—so it behaves nicely everywhere.
1. Domain: Polynomials are defined for all real x. Thus $D_f = \mathbb{R}$.	You can plug **any real number** into the formula; there are no forbidden inputs.
2. Parity / symmetry: $f(-x) = (-x)^4 - 10(-x)^2 + 9 = x^4 - 10x^2 + 9 = f(x)$. Hence **even.**	Replacing x by $-x$ doesn't change the value. The graph is **mirror-symmetric about the y-axis.** Understanding the right-hand side is enough—the left is its mirror.
3. Factorization setup: Let $t = x^2$. Then $f(x) = t^2 - 10t + 9$.	The expression depends on x^2, not on x directly. Treating x^2 as a single variable t reduces the problem to a familiar quadratic.
4. Zeros (solve $f(x) = 0$): $x^4 - 10x^2 + 9 = 0 \iff t^2 - 10t + 9 = 0$. Discriminant $D = 10^2 - 4 \cdot 1 \cdot 9 = 64$. Roots in t: $t_{1,2} = \frac{10 \pm \sqrt{64}}{2} = \frac{10 \pm 8}{2} \Rightarrow t_1 = 1,\ t_2 = 9$. Back-substitute $t = x^2$: $x^2 = 1 \Rightarrow x = \pm 1$; $x^2 = 9 \Rightarrow x = \pm 3$.	**Zeros** are the x-values where the graph crosses the x-axis (output $= 0$). There are **four crossing points:** $-3, -1, 1, 3$. These will break the x-axis into sign regions.
5. y-intercept: $f(0) = 0^4 - 10 \cdot 0^2 + 9 = 9$. Point: $(0, 9)$.	Where the graph meets the vertical axis: at height 9.
6. Full factorization & sign skeleton: $f(x) = (x^2 - 1)(x^2 - 9) = (x - 1)(x + 1)(x - 3)(x + 3)$. Ordered zeros: $-3 < -1 < 1 < 3$. Sign by intervals (checking any test point in each): $f > 0$ on $(-\infty, -3) \cup (-1, 1) \cup (3, \infty)$; $f < 0$ on $(-3, -1) \cup (1, 3)$.	Factor form makes sign-checking easy: a product is positive if an even number of factors are negative and negative if an odd number are. Result: **positive far left/right and near 0**, **negative between** $(-3, -1)$ and $(1, 3)$.
7. First derivative: Use $(x^n)' = nx^{n-1}$. $f'(x) = (x^4)' - 10(x^2)' = 4x^3 - 20x = 4x(x^2 - 5)$.	The derivative gives the **slope:** $f'(x) > 0$ means rising, $f'(x) < 0$ means falling, $f'(x) = 0$ means horizontal (candidates for peaks/valleys). The factored form will simplify sign analysis.
8. Critical points (solve $f'(x) = 0$): $4x(x^2 - 5) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{5} \approx \pm 2.236$.	Where the slope is zero, the graph flattens; these are the only candidates for local maxima/minima.
9. Function values at critical points: $f(0) = 9$. For $x = \pm\sqrt{5}$: $f(\pm\sqrt{5}) = (\sqrt{5})^4 - 10(\sqrt{5})^2 + 9 = 25 - 50 + 9 = -16$.	The center point is at height 9 (a peak), and the two side points are at -16 (deep valleys).
10. Monotonicity (sign of f'): Key cut points: $-\infty, -\sqrt{5}, 0, \sqrt{5}, \infty$. Pick simple test values:	Reading slope by zones gives the **overall shape:** left to right the curve goes **down → up → down → up.** Those up/down switches pinpoint the peak and the valleys.

FIgure1: Excerpt from the dual-perspective function analysis

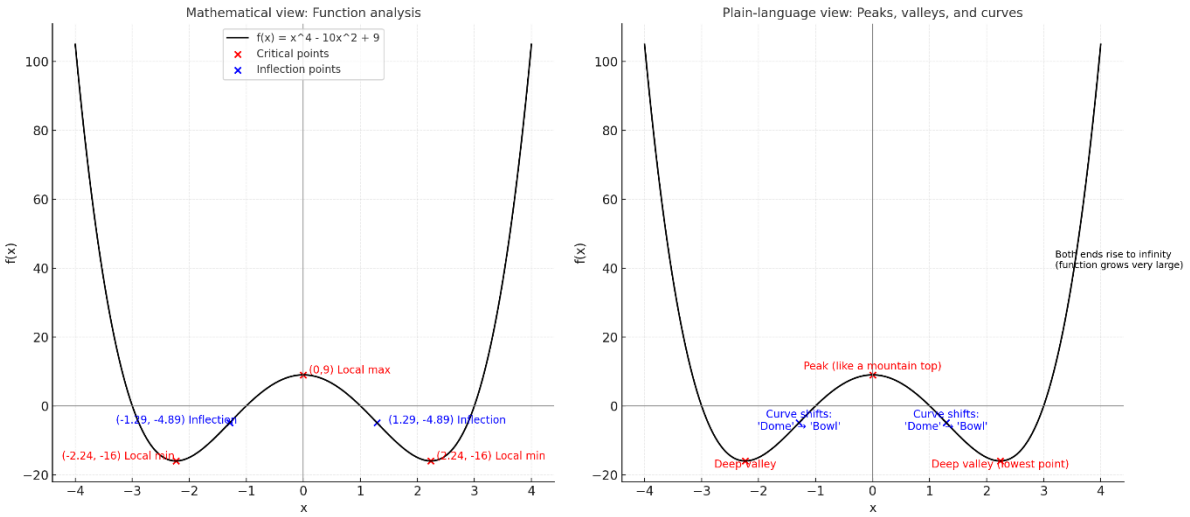

Figure 2: Dual representation of the function $f(x)=x^4 - 10x^2+9$

Thus, it can be concluded that one of the greatest advantages of AI-based systems is not only their ability to provide solutions to problems but also their capacity to function as step-by-step, explanatory mentors throughout the learning process. A student may enter an arbitrary function, for instance $f(x)=x^4-10x^2+9$, and the AI will automatically guide them through all the stages of function analysis. Rather than presenting only the final result, it provides short, targeted explanations at each step: *We differentiate here because*

this tells us whether the curve is increasing or decreasing. In this way, the student realizes that differentiation is not a magic trick but a logical operation with a clear purpose at every stage. The system thus functions like a patient teaching assistant, capable of rephrasing explanations whenever the student fails to grasp them at first [6, 14].

A second important didactic element is the dual-view representation, in which the learner is presented simultaneously with the *formal mathematical language* and the *plain-language explanation*. On the left-hand side, one sees the formal derivation, such as: f'(x)=0⇒ critical point, while on the right-hand side, one reads the everyday interpretation: *Here the curve is flat, as if you had reached the top of a hill on your bicycle.* This approach is particularly valuable because it helps students connect abstract symbols with everyday experience, thereby fostering deeper conceptual understanding. Educational psychology research suggests that such dual representation—formal and intuitive—strongly enhances both conceptual learning and long-term retention in memory [17].

The third dimension is interactive visualization (see Figure 3), in which the graph of the function is brought to life visually. A student may click on a point, and the AI automatically indicates: *This is a local maximum, because the curve rises before and falls after this point.* Beyond mathematical properties, such visualizations can also be enriched with economic explanations, for instance identifying a local maximum as a *profit peak*, a global minimum as a *cost minimum*, or an inflection point as the *onset of diminishing returns*. In this way, learners are encouraged to connect abstract calculus-based reasoning with concrete economic interpretations. The system can highlight zeros, extrema, and inflection points, allowing students to receive reinforcement not only through numerical results but also through conceptual and visual experience. The effectiveness of visual and interactive learning tools has been widely confirmed by empirical research, showing that active engagement and multimodal representation—particularly when combined with domain-specific applications—significantly enhance learning outcomes [18, 7].

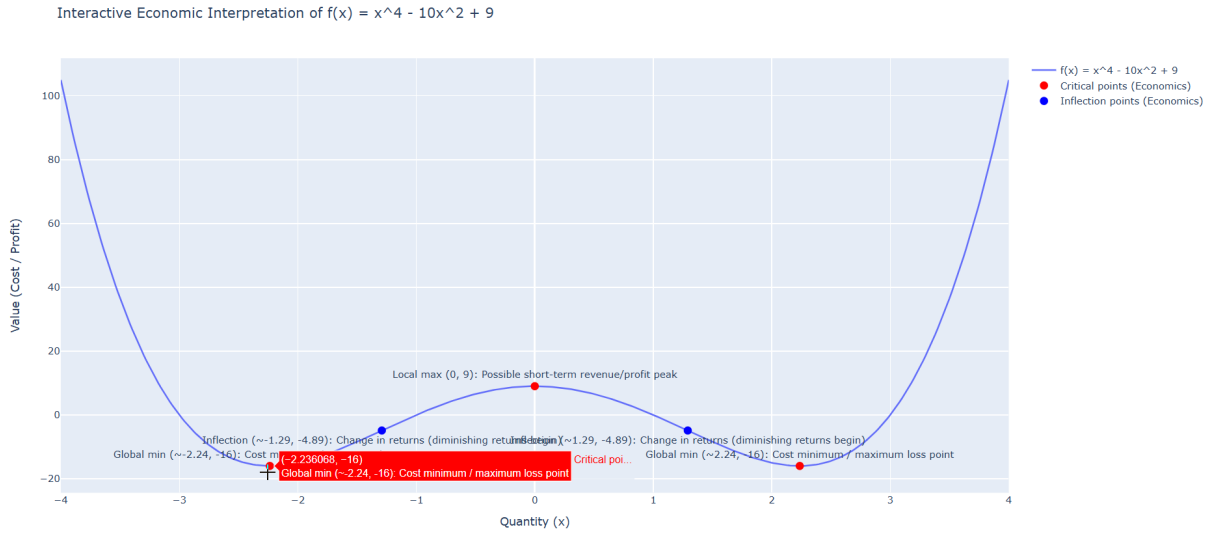

Figure 3: Interactive visualization with economic explanations

As previous studies have demonstrated, AI is capable of generating tasks of varying levels of difficulty within a given domain, thereby enriching the learning process [19, 20, 21]. Students can set the desired difficulty level, and the AI provides a corresponding function to analyze. If learners encounter difficulties, they may request hints instead of receiving the full solution. For example: *First, try to identify the domain of definition!* Such scaffolding is closely aligned with constructivist pedagogical principles, which

emphasize that learning is most effective when students actively construct their own knowledge while receiving guided support [22, 23].

Finally, learning motivation can be reinforced through gamification. AI can engage students with small challenges: *Where is the valley of the function?* or *Find the point where the curve changes its shape!* Correct answers may be rewarded with points, leaderboards, or visual incentives. The advantages of gamified approaches are highlighted by numerous studies, which demonstrate that playful elements enhance motivation, persistence, and overall learning experience [25, 26].

Taken together, step-by-step mentoring, the dual-view approach, interactive visualizations, personalized practice, economic applications, and gamification form a complex learning environment that simultaneously builds on logical-formal reasoning and intuitive-experiential understanding. For students, this does not only facilitate better mastery of mathematics but also enables the application of what they have learned to real economic and life situations.

Discussion

From Peaks to Valleys: How ChatGPT Combines Mathematical Rigor and Intuitive Metaphors in Function Analysis

The findings of this study highlight not only ChatGPT's ability to execute the formal steps of function analysis with accuracy but also its potential to act as a pedagogical bridge between abstract mathematics and applied economic reasoning. For students in economics, the ability to interpret mathematical functions extends far beyond technical correctness; it is deeply tied to decision-making in real-world contexts such as minimizing production costs, maximizing revenues, or identifying the profit-maximizing level of output.

One of the central challenges faced by students is the transition from mechanical procedures—such as computing derivatives or solving equations—to conceptual interpretation, which requires an understanding of what these mathematical results mean in practice. For instance, recognizing that a local maximum corresponds not only to a point where the first derivative equals zero, but also to the point of greatest profit in a profit function, is a leap from symbolic manipulation to economic reasoning. ChatGPT's dual explanatory style addresses precisely this pedagogical gap.

By describing maxima as mountain peaks and minima as valleys, the system provided learners with intuitive visualizations that made abstract results easier to grasp. When these metaphors are combined with the formal symbolic results, students can align two perspectives: the rigorous mathematical logic and the intuitive economic meaning. In analyzing a cost function, finding the minimum reveals the most efficient production level; in examining a revenue function, the maximum shows the level of sales that yields the greatest revenue; while the maximum of a profit function directly identifies the most profitable level of output. AI not only calculates these points but also immediately translates them into the language of economics: *This maximum point represents the profit maximum—this is the optimal level of production.* Thus, learners clearly see that function analysis provides a concrete foundation for real-world decision-making [16, 24]. Moreover, the ability of ChatGPT to explain inflection points—transitions from convex to concave behavior—through imagery such as the change from a *dome* to a *bowl* also carries economic significance. In economics, inflection points can represent critical thresholds where marginal effects change direction, such as the point at which diminishing returns set in (see Figure 4). This dual interpretation, both mathematical and metaphorical, strengthens the learner's capacity to connect calculus-based reasoning to economic intuition.

Step-by-step function analysis with economic interpretation

Megosztás

Formal mathematics (step by step)	Plain-language explanation (what/why)	Economic interpretation (so what?)
0. Problem: Analyze $f(x) = x^4 - 10x^2 + 9$ (quartic polynomial).	A rule that turns any x into $f(x)$. Polynomials behave well.	Same math workflow you'll use for **cost/revenue/profit** functions.
1. Domain: Polynomials are defined for all real x. So $D_f = \mathbb{R}$.	You can plug in **any** real number—no restrictions.	Real decisions often allow any feasible quantity q. Domain tells you where analysis applies.
2. Parity / symmetry: $f(-x) = f(x) \rightarrow$ **even.**	Mirror symmetry around the y-axis; left side repeats the right.	Symmetry can **halve your work**; in econ, symmetric cost/utility shapes simplify reasoning.
3. Factorization setup: Let $t = x^2$. Then $f = t^2 - 10t + 9$.	Treat x^2 as one variable to make it a simple quadratic.	Re-expressing problems to simpler forms is key in optimization (e.g., substitutions to linearize).
4. Zeros (solve $f = 0$): $t^2 - 10t + 9 = 0 \rightarrow t = 1, 9$. Back: $x = \pm 1, \pm 3$.	Zeros are where the graph crosses the x-axis: $-3, -1, 1, 3$.	Crossing points often mark **thresholds** (e.g., break-even quantities where profit $= 0$).
5. y-intercept: $f(0) = 9 \rightarrow$ point $(0, 9)$.	Where the graph hits the vertical axis.	Baseline value at "no output" can echo **fixed cost / initial utility**, etc.
6. Full factorization & signs: $f = (x^2 - 1)(x^2 - 9)$. Sign: $f > 0$ on $(-\infty, -3) \cup (-1, 1) \cup (3, \infty)$; $f < 0$ on $(-3, -1) \cup (1, 3)$.	Factor form makes it easy to see where the function is positive/negative.	Positive vs. negative regions can mean **profit vs. loss** zones or feasible vs. infeasible payoffs.
7. First derivative: $f'(x) = 4x^3 - 20x = 4x(x^2 - 5)$.	Derivative = **slope:** $+$ rising, $-$ falling, 0 flat.	In econ, $f'(q)$ is **marginal:** marginal cost/revenue/profit guiding adjustments.
8. Critical points: Solve $f'(x) = 0 \rightarrow x = 0, \pm\sqrt{5} \approx \pm 2.236$.	Flat spots—candidates for peaks/valleys.	Where **optima** can occur (profit max, cost min). You must test/classify them.
9. Values at critical points: $f(0) = 9$; $f(\pm\sqrt{5}) = -16$.	Center is high (9), sides are low (−16).	Identifies **best/worst outcomes:** e.g., minimum cost or maximum profit levels.

Figure 4: Mathematical, intuitive, and economic interpretations of function analysis results

This alignment reduces cognitive load, as learners are not left to infer the meaning of symbols independently but are guided by accessible, everyday analogies.

An additional implication is the potential role of ChatGPT as a scaffolding tool in the learning process. Students often require repeated explanations delivered at different levels of abstraction. A human instructor may not always have the time to provide highly individualized feedback, whereas ChatGPT can generate both precise mathematical detail and accessible analogies on demand. Importantly, this does not imply that AI replaces the teacher. Rather, it functions as a supplementary resource, allowing instructors to shift their focus toward higher-order skills such as critical thinking, evaluation, and synthesis.

A particularly important dimension is the integration of economic applications, through which students can experience that function analysis is not merely an abstract mathematical exercise but also a tool for grounding real economic decisions. In analyzing a cost function, finding the minimum reveals the most efficient production level; in examining a revenue function, the maximum shows the level of sales that yields the greatest revenue; while the maximum of a profit function directly identifies the most profitable level of output. AI not only calculates these points but also immediately translates them into the language of economics: *This maximum point represents the profit maximum—this is the optimal level of production.* Thus, learners clearly see that function analysis provides a concrete foundation for real-world decision-making [16, 24].

In sum, the results demonstrate that ChatGPT's greatest pedagogical value lies not in its flawless execution of symbolic procedures—which conventional software could also perform—but in its capacity to explain mathematical ideas on multiple levels simultaneously. This ability directly supports economics education, where students must not only perform calculations but also translate them into meaningful interpretations that inform strategic decision-making.

Why Learn Function Analysis if AI Can Do It? The Role of Students and Teachers

Seeing ChatGPT's solutions and explanations naturally raises a pressing question: if AI can perform all of these tasks, why should students still learn them? And further, why do we still need teachers? With the rapid advancement of generative AI, these questions are becoming part of everyday academic discourse. At first glance, one may wonder: how is ChatGPT different from a computer algebra system, which has long been available for symbolic computation? To address this, we use function analysis as a lens to explore two fundamental questions:

- If AI can calculate everything, why should I learn it?
- Why do we still need teachers?

Indeed, ChatGPT and similar AI systems can instantly compute derivatives, zeros, or maxima. Yet, there are several reasons why human learning and teaching remain indispensable:

- **Numbers alone are not enough.**
  It is essential to understand what a numerical result actually means in practice. For instance, if the output states: *the profit maximum is at 20 units*, an economist must be able to interpret this: producing 19 units means leaving money on the table, while producing 21 units results in losses.
- **Context and decision-making remain ours.**
  A computer does not know what we want to optimize—price, cost, or profit. It is up to us to identify which function is relevant for the problem at hand, and why it needs to be analyzed.
- **Critical thinking is indispensable.**
  If we blindly accept machine outputs, we become vulnerable. By understanding the underlying concepts, we can judge whether a result is plausible or whether it may be flawed due to an incorrect model, formula, or assumption.

In short, while AI can perform the calculation, it is up to us to interpret and apply the results. The core of learning is not the computation itself but grasping the logic behind economic decisions and cultivating the ability to critically evaluate outcomes.

When it comes to the role of teachers, although AI can explain, calculate, and visualize, a teacher is far more than a computer in explanatory mode. A teacher's responsibilities include:

- **Selecting content.** It matters which examples are chosen and in what order they are presented.
- **Motivation.** AI can provide answers, but it cannot inspire, praise, or sense when a student is losing focus.
- **Creating connection.** Teachers recognize group dynamics and know when to slow down, introduce playful elements, or adjust their approach.
- **Transmitting values.** Learning is not only about numbers, but also about perseverance, logical reasoning, and collaboration.
- **Asking questions.** True learning is not simply about receiving answers but about learning to formulate the right questions. Teachers guide students toward asking and answering meaningfully.

In summary, a teacher is simultaneously a mentor, coach, and human support system. AI may serve as a powerful tool, but without teachers, learning would often be directionless and impersonal. Teachers provide not only explanations but also leadership, motivation, questioning, and human connection—none of which can be replaced by an algorithm.

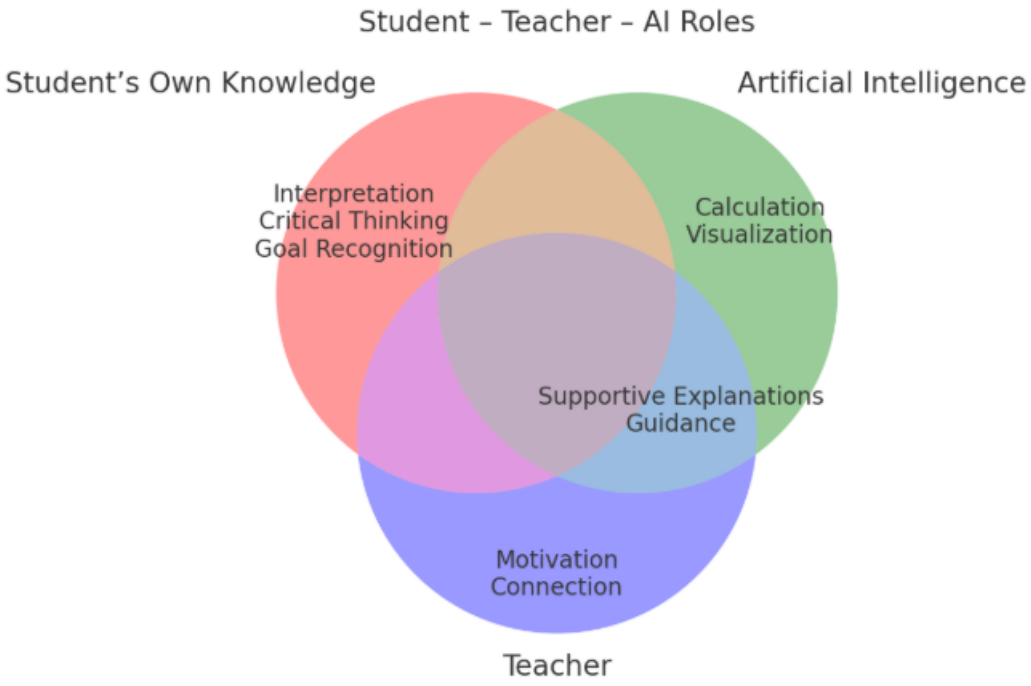

Figure 5. Student, Teacher, and AI roles.

Figure 5 illustrates the significance of the three-way collaboration among teacher, student, and AI. The teacher provides the framework, sets the goals, and motivates; the student actively participates, asks questions, and reflects; while AI offers rapid calculations and alternative explanations. The most effective learning environment emerges from the synergy of these three elements.

Conclusions

The results of this investigation demonstrate that ChatGPT has considerable potential as a complementary tool in the teaching and learning of function analysis, particularly within economics education. On the one hand, the model is capable of carrying out the technical aspects of the task—such as identifying the domain, calculating derivatives, determining critical points, and classifying intervals of convexity and concavity—with a level of accuracy comparable to traditional computer algebra systems. On the other hand, its distinguishing feature lies in its explanatory power: ChatGPT is able to supplement the formal mathematical solution with intuitive, plain-language interpretations, such as describing local maxima as *mountain peaks* or inflection points as the transition from a *dome* to a *bowl*. These analogies make abstract mathematical structures more tangible and relatable, thereby reducing cognitive barriers that students—especially those without a strong mathematical background—often encounter.

For economics students, this dual representation is of particular significance. Function analysis is not merely an academic exercise but a foundation for understanding real-world decision-making problems, such as minimizing production costs, maximizing revenues, or identifying profit-maximizing output levels. When the AI highlights both the mathematical logic and its economic implications, students can more easily transfer abstract knowledge into applied contexts. This connection between mathematics and economics strengthens not only procedural competence but also the ability to reason critically about economic trade-offs and optimization problems.

At the same time, the findings underscore that ChatGPT should not be regarded as a replacement for traditional teaching. While the system can calculate and explain, it cannot fulfill essential pedagogical

roles: selecting appropriate examples, scaffolding the progression of learning, motivating students, recognizing affective and social cues, or instilling academic values such as persistence, collaboration, and critical reflection. Without teacher guidance, there is a risk that students may rely on the AI as a black box, accepting results uncritically without developing the necessary judgment to evaluate their validity. This highlights the need for pedagogical frameworks in which AI is positioned not as an autonomous tutor but as a supportive assistant integrated into a teacher-led learning environment.

The broader implication is that the future of mathematics education—and by extension economics education—lies not in prohibiting the use of generative AI tools, but in cultivating their conscious, ethical, and reflective integration. By leveraging AI for mechanical tasks such as computation and visualization, educators can redirect classroom focus toward higher-order cognitive skills, including interpretation, contextualization, and critical decision-making. If integrated thoughtfully, ChatGPT and similar systems may help students not only acquire mathematical skills but also learn to apply them in navigating the complexities of real-world economic problems.

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